THE SELF-SIMILARITY OF GEODYNAMIC PROCESSES

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АВТОМОДЕЛЬНОСТЬ ГЕОДИНАМИЧЕСКИХ ПРОЦЕССОВ

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The rocks that compose the Earth's crust have two characteristic properties that, even though familiar, have not attracted due attention. The one is *discreteness*. The Earth's crust consists of units of different scales, ranging from minutest sand particles to continental plates, smaller units being as it were embedded in larger ones (Fig. 1).

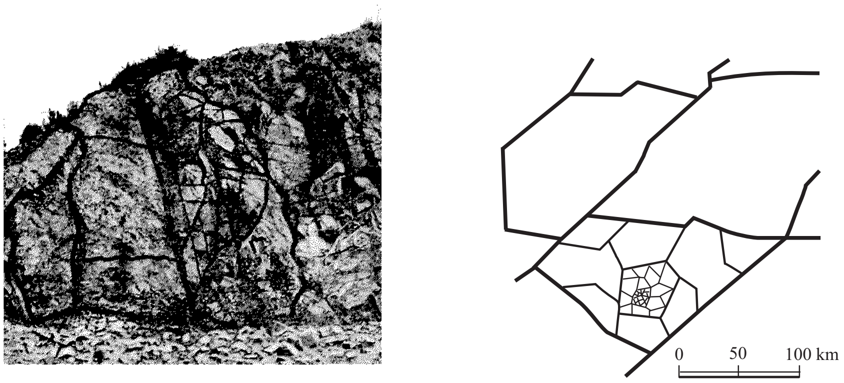

Fig. 1. Wall of a quarry excavated for rock extraction with a well-defined blocky structure (left) and a scheme of rock structure in which large blocks are smaller systems that are as it were embedded in one another (right)

The study of distributions of rock units over size showed that, in contrast to what was previously held to be the case, all these distributions are polymodal, the modes being practically independent of the physico-chemical properties of the rock. It has also emerged that not all unit sizes are equally probable, some of them being more frequent than the others. It was further found that these, more frequent, "dominant" sizes have the remarkable property of forming a hierarchical sequence. The sequence can be approximately described by a geometric progression whose common ratio K is nearly constant. The common ratio is not only independent of the physico-chemical properties of the rock, but also remains unchanged whatever the method of producing the units, whether it is by natural cracking, fragmentation by underground explosions or pulverizing by mills and so on (Figs. 2, 3, 4). The quantity K varies between 2 and 5 for an enormous range of size (Fig. 5). On the average, one has $K = 3.5 \pm 0.9$.

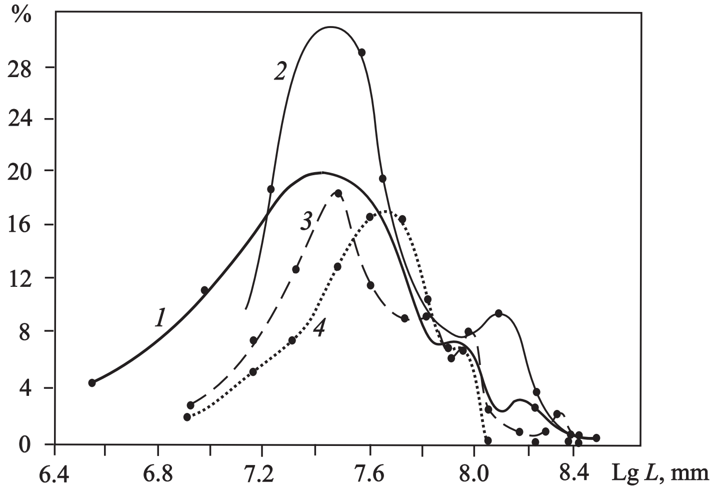

Fig. 2. Distribution of crustal blocks by size for various regions:
1 – Asia Minor, *2* – (former Soviet) Central Asia, *3* – California, *4* – blocks bounded by geologic faults as identified by helium surveys; the mean value is $K = 3.2$

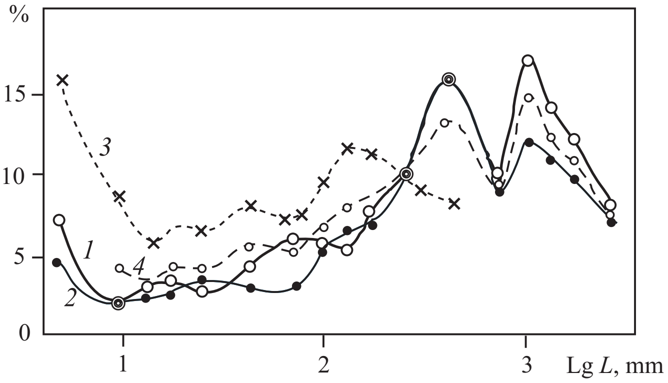

Fig. 3. Distribution of rock fragments by size as crushed by underground explosions of various yield and nature:
1 – nuclear blast of yield 0.42 kt, *2* – chemical blast of yield 0.02 kt, *3* – chemical blast of yield 0.7 kt, *4* – nuclear blast of yield 5.4 kt; the mean value is $K = 2.9$

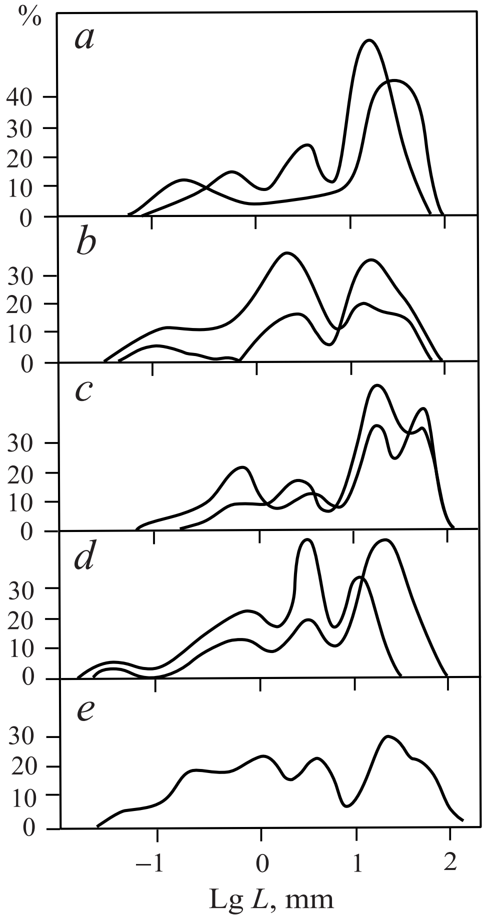

Fig. 4. Distribution of rock units by size given by a granulometric analysis of Baikal rocks (V.G. Simonov's data):

a – effusives, *b* – granite, *c* – sandstone, *d* – quartz, *e* – mean disytribution; the mean value is $K = 3.7$

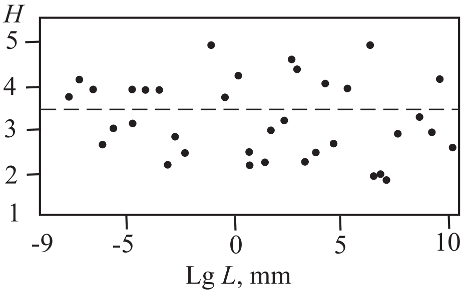

Fig. 5. Variation of K in the size range from a few hundredths of micrometer to hundreds of thousands of kilometer. The dashed line stands for the mean $K = 3.5 \pm 0.9$

The statement that K is constant is statistical in character, like the notion of a "dominant" size. It would be more correct to speak about a "dominant" interval of unit size. The existence of a "statistical" constancy of K provides evidence of *self-similarity* in the production of units, their similarity and independence of size, physico-chemical properties, and of the way the rock units have been produced.

The other important property of rocks is that they are subject to *permanent oscillatory motion* in a wide frequency range. We know of microseism oscillations at frequencies of the hertz range and long-period free oscillations of the Earth. In recent years researchers have become interested in acoustic and ultrasonic noise in the crust. High frequency elastic waves rapidly decay; consequently, the fact that they are being recorded shows them to be generated in a diffuse manner throughout the lithosphere due to some external cause. Bearing in mind what has been said above on the discrete structure of rock, it would be natural to regard its oscillations in a variety of spectral ranges to be free oscillations of different-sized constituent units.

A mathematical model that can describe processes involved in the discretization of rocks is still nonexistent. Some qualitative characteristics of such a model are described below. The Earth is conceived as an open, multicomponent system that is capable of receiving and reworking (transporting and transforming) the energy coming into it from outside. The same properties are shown by the subsystems that compose it: the lithosphere, the crust, rocks and rock blocks. The important fact is that the external sources of energy - the Sun, the gravity field and, for the subsystems - heat flow from the Earth's interior, tectonic movements etc. - remain practically constant over time spans observable in human history.

Energy coming into an open system is reworked there, the reworking mechanism being possibly of rather different physico-chemical character (mechanical motion, polymorphic changes etc.). Here, we will restrict ourselves to mechanics. Imagine a system consisting of elastic units, blocks, that can receive energy from outside and exchange it among themselves. Some blocks will receive energy from outside and lose stability, emitting some of the stored energy in the form of elastic waves which are in turn absorbed by adjacent similar-sized blocks. Such an evolution of the system will affect the configurations of the constituent blocks. They are being displaced relative to one another and experience a kind of regrouping, creating a structure that is suitable for reworking the incoming energy. However, even after the structure has been created, the energy is fluctuating, coming into the sytem from outside and being emitted by it to outer space. A kind of dynamic equilibrium settles down. The energy exchange never terminates completely, because part of system energy is always being dissipated, hence the elements of the system never stop oscillating.

Numerous observations corroborate the fact that rocks always oscillate in a wide frequency range (hundreds of kilohertz to ten thousandths hertz or less: acoustic emission, microseisms, tidal motions etc.). It is important to point out that the practical constancy of energy flux coming to the Earth from outside is responsible for the circumstance that the properties of the system Earth itself and of the constituent rocks remain practically unchanged during all processes that are going on in it.

The above mechanism of self-organization for a system consisting of rock blocks is not the only possible one. Polymorphic phase transitions which affect structural and chemical properties of the system also belong to this category of phenomena. Different mechanisms may probably be operative simultaneously. These questions call for a special study, however.

So far we have been concerned with self-organization in rocks alone. There is evidence enough however that self-similar ordering in the structure of materials occurs in many different systems. To take one example, the structure of silica glass (Fig. 6) is remarkably like that of blocky rock structure (see Fig. 1). There is the notable fact that the distribution of solar system bodies (planets, satellites, asteroids) by size also follows a hierarchical sequence with K equal to 3.5.

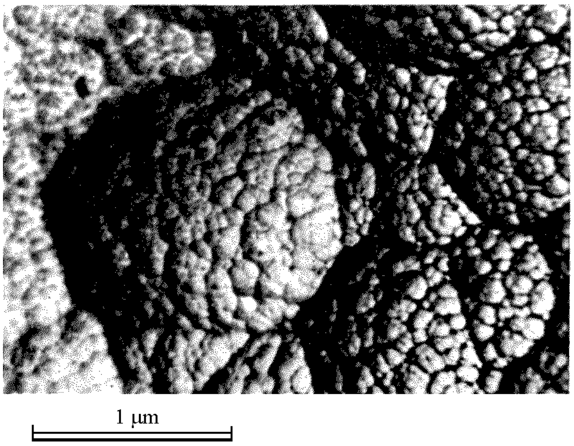

Fig. 6. Electron microscope image of a silica glass microsection. Three globule sizes are clearly seen; the mean value is $K = 3.8$

All this suggests that self-similarity is a typical property of many natural and man-induced self-organizing processes in solids, and probably fluids. If the hypothesis is valid, it can then be asserted that time characteristics of these processes must exhibit discreteness as well. We have tried to look from this standpoint at the data presented by S.L. Afanasiev in his table "Levels of organization, geological cycles and their duration", supplementing it with ratios of adjacent cycle durations T.

It appears from Table 1 that the ratios $T_i : T_{i+1}$ vary between 2.4 and 4.7 at all organization levels, with the exception of the "layers" row. The "layers" ratio is an outlier, being greater than 8. When one recalls however that 600 years is one of the most familiar cycles in the variation of the geomagnetic field, and inserts class 12a of $0.6 \cdot 10^3$ yr duration into the "layers" level, then the outlying ratio value can be replaced with two ones equal to 3 and 2.7. The insertions are bracketed in Table 1. The mean $T_i : T_{i+1}$ ratio then turns out to equal 3.7 ± 0.6, i.e., is practically identical with the mean $K = 3.5 \pm 0.9$.

TABLE 1. Levels of organization, geological cycles and their duration

Level of organization	Time span	Cycle	Mean duration (T), year	Class	Subsystems	Ratio $T_i : T_{i+1}$
Shell	History of Earth	Mega	$4.8 \cdot 10^9$	1	Earth's crust	3.2
	Megachron		$1.5 \cdot 10^9$	2	Shell of crust	2.4
	Eon		$6.5 \cdot 10^8$	3	Structural stage	3.2
	Era		$2.15–1.90 \cdot 10^8$	4	Structural substage	3.5
Formations	Period	Macro	$58 \cdot 10^6$	5	System	4.1
	Epoch		$14 \cdot 10^6$	6	Series	3.8
	Age		$3.7 \cdot 10^6$	6	Suite	4.6
Fascies	Phase	Meso	$0.8 \cdot 10^6$	8	Subsuite	4.2
	Term		$190 \cdot 10^3$	9	Member	4.7
	Geominute		$40 \cdot 10^3$	10	Submember	4.2
Layers	Episode	Micro	$9.5 \cdot 10^3$	11	Packet	5.3 (5.3)
	Polysecond		$1.8 \cdot 10^3$	12	Complex polylayer	8.2 (3.0)
			$(0.6 \cdot 10^3)$	(12a)	Paleomagnetism	(2.7)
	Geosecond		$0.22 \cdot 10^3$	13	Simple polylayer	3.7 (3.7)
Beds	Secular nanocycle	Nano	60.0	14	Bed	4.6
	Solar nanocycle		13.0	15	Complex bed	3.7
	Geotriplets		3.5	16	Simple polybed	3.7
	Year		1.0	17	Pair of beds	–

That the self-organization in rocks caused by energy coming from without is self-similar is also corroborated by determinations of the time Δt elapsed since the appearance of the earliest precursors before a future earthquake until the time of the main event itself. We used numerous determinations of Δt for the precursors that take place in the earthquake source volume (variation of seismic velocities, anisotropy, electric conductivity etc.) to plot them as a function of seismic energy E_s (Fig. 7). It turns out that the relationship can be fitted by

$$\lg\, \Delta t = \frac{1}{3} \lg\, E_s - 7,$$

here Δt is in years and E_s in ergs.

Recalling that earthquake energy E_s is connected with the average earthquake source size L_0 as

$$\lg\, E_s = 3 \lg\, L_0 + 3,$$

one can rewrite the above relationship as

$$\lg\, \Delta t = 3 \lg\, L_0 - 6,$$

where L_0 is in centimeters and Δt in years.

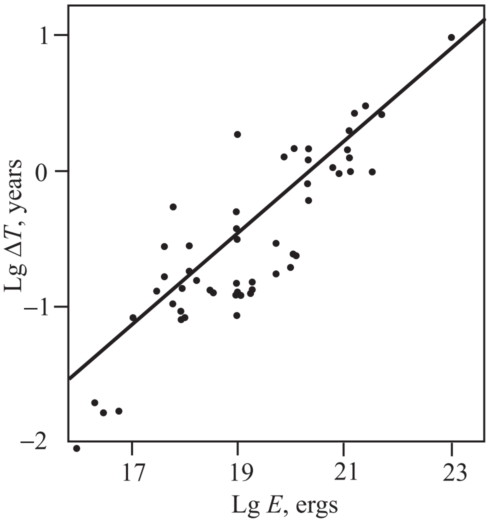

Fig. 7. Precursor time as a function of earthquake energy. The straight line stands for $\lg \Delta t = \frac{1}{3} \lg E_s - 7$

If self-similarity in the processes of structure reshaping exists in such a broad range of size as pointed out above, it would seem that the relation for determining precursor times can also be used to find delay times of low magnitude earthquakes and failure in rock specimens during experiments in the laboratory. Table 2 shows theoretical and observed Δt for low magnitude earthquakes and for delay times of failure in rock specimens under presses.

TABLE 2. Times of appearance for precursors of low magnitude earthquakes and delays of failure in laboratory rock specimens

Laboratory			Earthquake		
Size, cm	Δt, s		Size, cm	Δt	
	Theory	Observation		Theory	Observation
0.1	3	1	$2 \cdot 10^3$	20 hours	–
1.0	30	30	10^4	4 days	–
10.0	300	100	$4 \cdot 10^4$	15 days	30 days
100.0	1500	–	$2 \cdot 10^5$	60 days	40 days
			10^6	12 months	13 months

One notes that experimental and theoretical determinations of Δt are in very satisfactory agreement, considering the probabilistic nature of earthquake processes and failure in specimens. One may therefore expect that the new "blocky" rock model is likely to be helpful in dealing with some other geophysical problems which do not fit into the framework of the mechanics of continua and linear elasticity.

Let us discuss a few more obvious cases where the new model of the geophysical medium can be used. It was pointed out above that the constancy of energy flux into an open rock system determines a dynamical invariability of its structure, hence of its properties. This equilibrium can in principle be disturbed by some extra energy coming from an external source. The source may be, e.g., a powerful vibrator installed at the ground surface or in a mine. Vibratory energy pumped into the rock will necessarily cause its restructuring which will be more serious the more energy is elastically absorbed by units of the system. One practical result from this operation may consist in changes of rock properties such as permeability, electrical conductivity etc. It is not ruled out that energy can be pumped into rock, apart from mechanical vibrations, also by exciting alternating electromagnetic fields in it. Lastly, structural changes in rock can be caused, not only by energy pumping, but also by exchange of mass and energy with the surrounding medium. To a certain extent the contemporary methods in use to pump water into oil-bearing formations to enhance oil production may be classified as the techniques for rock restructuring disussed above. One can probably conceive a broader problem of human interference in the mass-energy transport in rocks, to be achieved by various methods used to pump energy and material (fluids, including surface-acting ones) into rocks.

One recalls that the depth of mining, hence the cost of extraction, are rapidly increasing. Therefore, mining methods such as leaching, which are amenable to control using energy-mass transport, will certainly gain in importance.

It goes without saying that the problem is not easily solvable, technically speaking. A reasonable choice of the power and frequency output of the vibrator will be required, not to mention the determination of the time of energy pumping required among many other things. However, the few experimental data available inspire some hope of success. For instance, the new model allows an understanding of the processes that affect the properties of the medium due to the action of external physical fields. We have long known cases where the discharge of oil wells, the amount and composition of the gases which are released from rock changed by mechanical vibrations, e.g., microseisms. Special studies are made of acoustic emission arising during the passage of seismic waves in rocks, as well as of several electromagnetic phenomena that cannot be understood using the linear elastic model of continua. We think that the new model can provide a physical basis for all these effects which are traditionally treated as nonlinearity.

There would probably be no sense dwelling on such distant perspectives, so we will restrict ourselves for the moment to problems that can certainly be solved. One of these is the new method for modeling geotectonic processes. So far the modeling has been based on the concepts of continuum and linear elasticity. Although these concepts have helped toward solving some problems in geostatics, the method has proved to be rather limited, similarly to the situation in seismotectonics. Modeling studies are at present popular in this country and abroad using discrete media which consist of specially manufactured particles and of natural sand.

To sum up, the new model of rock, which is part of much broader concepts concerning the properties of natural open systems that are capable of self- organization due to energy coming from external sources, can be used, not only to achieve a better understanding of geological and geophysical processes, but also to deal with a number of major problems in science and technology. It is in principle not ruled out that external methods for controlling the seismic process can be developed. At present, however, this feasibility can only be discussed as a distant perspective for which contemporary technical facilities are insufficient.

A special question concerns the range of size where the hierarchical distribution of material units is observed. The only thing to be said is that the range is enormous. In view of the hypothesis that the material of the Universe is fragmented when a galaxy is generated, the upper bound of the range becomes entirely indeterminate. The lower bound too is difficult to assess. Since the chemical properties of the material concerned do not affect the hierarchical sequence of dominant size, one should think that the lowest possible size is somewhere at the submolecular level. There is still much to be done for this issue to be clarified.

We conclude by calling once more the attention of the reader to the similarity of all the examples drawn from geology and geophysics. Self-similarity is perhaps one of the general properties common to the universe-wide process of self-organization of material.

A list of the papers by academician M.A. Sadovsky related to the hierarchical model of the geophysical medium

1. *M.A. Sadovsky.* On the natural divisibility of rocks // DAN SSSR. 1979. Vol. 247, N 4. P.829.
2. *M.A. Sadovsky.* The distribution of sizes of solid units // DAN SSSR. 1978. Vol. 269, N 1.
3. *M.A. Sadovsky, Sardarov S.S.* Subordination and similarity of geologic motions in connection with the natural divisibility of rocks // DAN SSSR. 1980. Vol. 250, N 4. P.846–848.
4. *M.A. Sadovsky, Bolkhovitinov L.G., Pisarenko V.F.* On discreteness in rocks // Fizika Zemli. 1982. N 12. P.3–18.
5. *M.A. Sadovsky, Pisarenko V.F., Rodionov V.N.* From seismology to geomechanics: a model of the geophysical medium // Vestnik AN SSSR. 1983. N 1. P.82–88.

6. *M.A. Sadovsky.* The hierarchical distribution of units of solid materials (in Russian). M. 1984. 20 p. (Prepr. B22809/IFZ AN SSSR).
7. *M.A. Sadovsky, Pisarenko V.F.* The seismic process in a blocky medium. M.: Nauka, 1991. P.96, (in Russian).
8. *M.A. Sadovsky* On models of the geophysical medium and of the seismic process // Earthquake prediction. Dushanbe: Donish Publ, 1984. N 4. P.268–273, (in Russian).
9. *M.A. Sadovsky, Golubeva T.V., Pisarenko V.F., Shnirman M.G.* Typical rock sizes and the hierarchical properties of seismicity // Izv. AN SSSR. Fizika Zemli. 1984. N 2. P.3–15.
10. *M.A. Sadovsky, Kedrov O.K., Pasechnik I.P.* On seismic energy and source volumes for crustal earthquakes and underground explosions // DAN SSSR. 1985. Vol. 283, N 5. P.1153–1156.
11. *M.A. Sadovsky, Pisarenko V.F.* Dependence of precursor time on earthquake size // DAN SSSR. 1985. Vol. 283, N 6. P. 1359–1361.
12. *M.A. Sadovsky* The self-similarity of seismic processes // Physical bases for predicting rock fracture of earthquakes. M.: Nauka, 1987. P.6–12, (in Russian).
13. *M.A. Sadovsky, Bolkhovitinov L.G., Pisarenko V.F.* Deformation in the geophysical medium and the seismic process. M.: Nauka, 1987. 100 p., (in Russian).
14. *M.A. Sadovsky, Kocharian G.G., Rodionov V.N.* On the mechanics of a blocky rock mass // DAN SSSR. 1988. Vol. 302, N 2. P.306–308.
15. *M.A. Sadovsky.* On the importance and meaning of discreteness in geophysics // Discrete properties of the geophysical medium. M.: Nauka, 1989. P.5–13, (in Russian).
16. *M.A. Sadovsky.* Faults and seismicity // DAN SSSR. 1989. Vol. 307, N 5. P.1089–1091.
17. *M.A. Sadovsky.* A blocky hierarchical model of rocks and its use in seismology // Experimental and numerical methods for the earthquake source physics. M.: Nauka, 1989. P.5–13, (in Russian).